Problem 16.7

[Computer] Make plots of the two triangular waves of Example 16.1 (page 686) at several closely spaced times and then animate them. Describe the motion. For the purposes of the plot you may as well take the speed c, the height of the triangle at time 0, and the half width of the base all equal to 1. Make your plots for lots of times ranging from $t = -4$ to 4.

Solution

The general solution to the initial value problem,

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty,\ -\infty < t < \infty \\ &u(x,0) = u_0(x) \\ &\frac{\partial u}{\partial t}(x,0) = 0, \end{aligned}$$

was found in Problem 16.6 to be

$$u(x,t) = \frac{1}{2}[u_0(x-ct) + u_0(x+ct)].$$

The function representing a triangle with a height and a half-width of 1 is

$$u_0(x) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}.$$

If $c = 1$, then

$$u(x,t) = \frac{1}{2}[u_0(x-t) + u_0(x+t)],$$

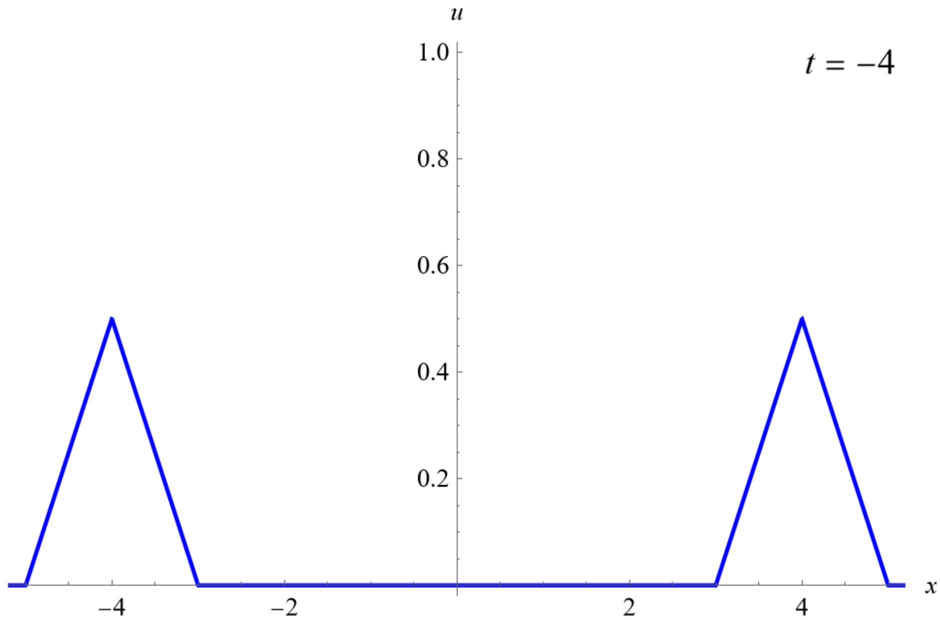
where

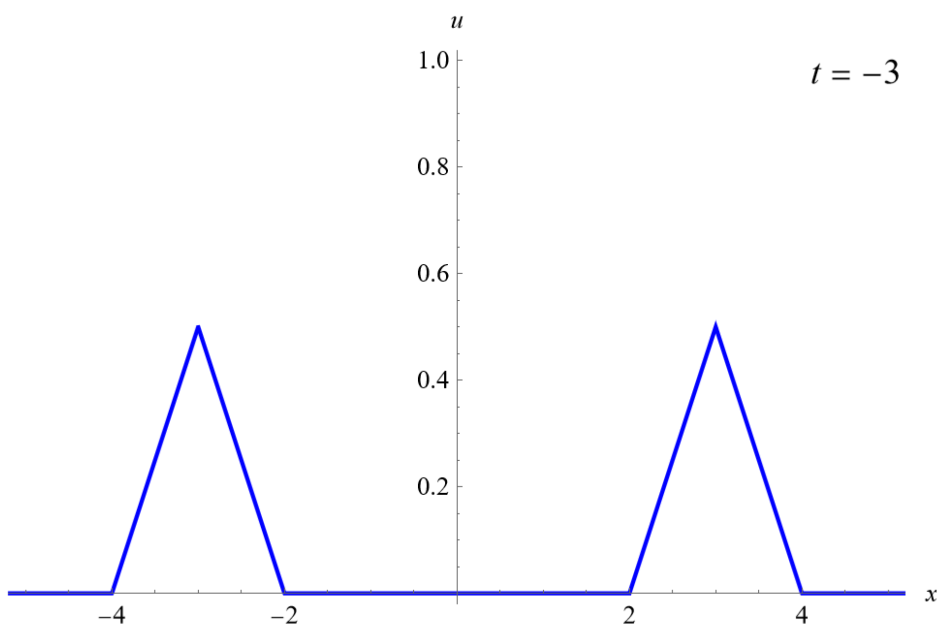
$$u_0(x-t) = \begin{cases} 1 - |x-t| & \text{if } |x-t| \leq 1 \\ 0 & \text{if } |x-t| > 1 \end{cases}$$

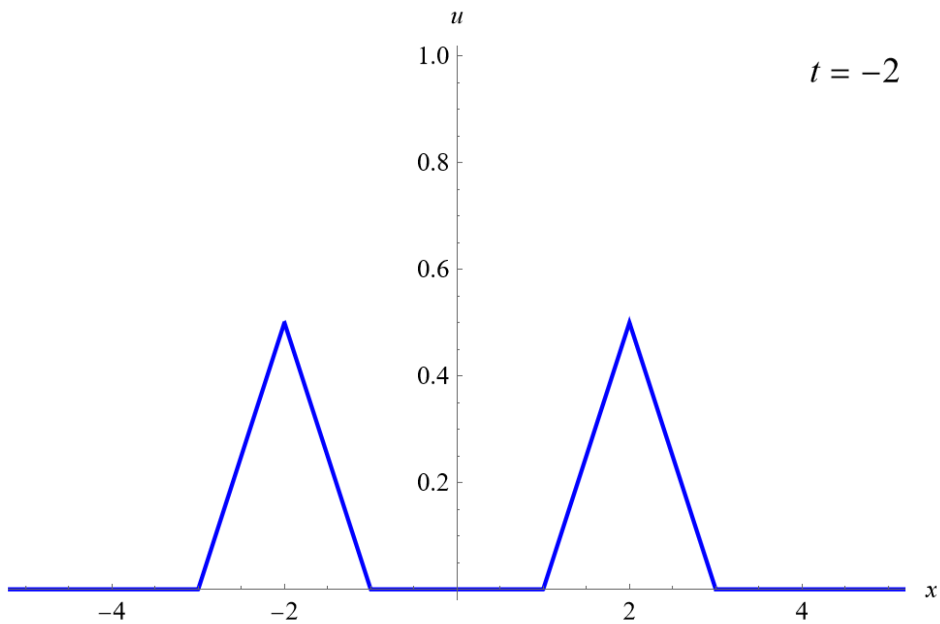
and

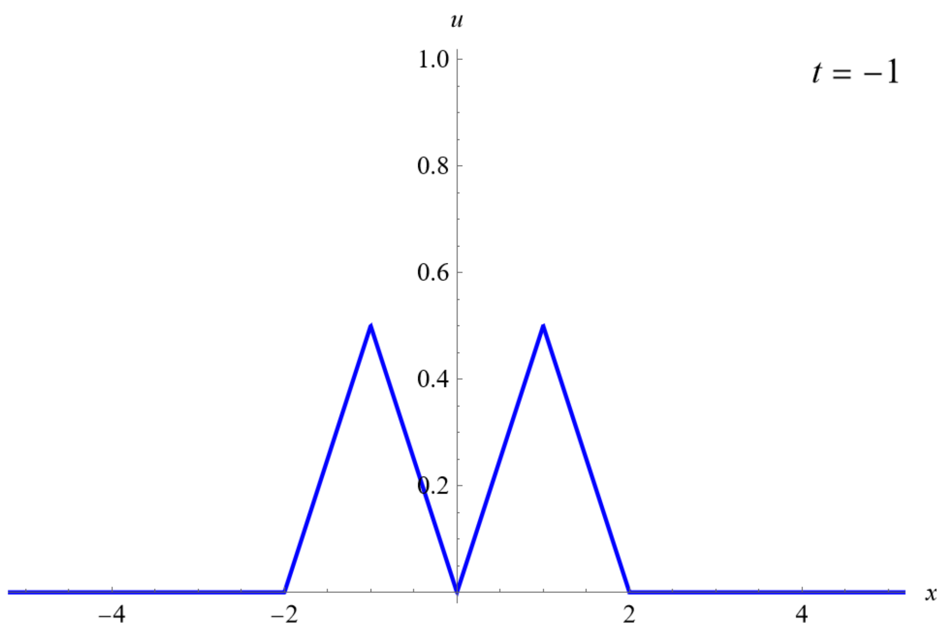
$$u_0(x+t) = \begin{cases} 1 - |x+t| & \text{if } |x+t| \leq 1 \\ 0 & \text{if } |x+t| > 1 \end{cases}.$$

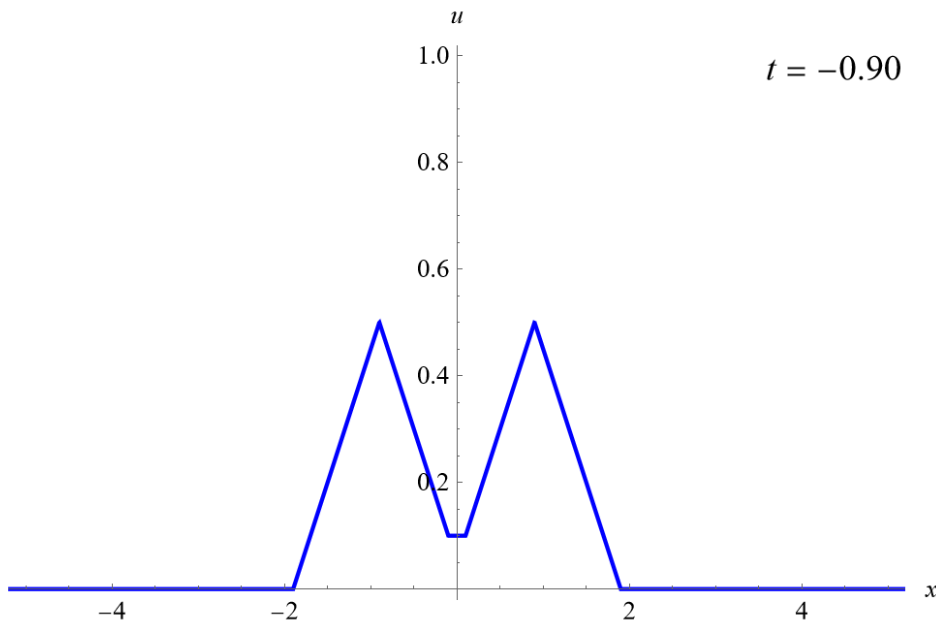
Below are computer-generated plots of $u(x,t)$ versus x at many values of t between -4 and 4.



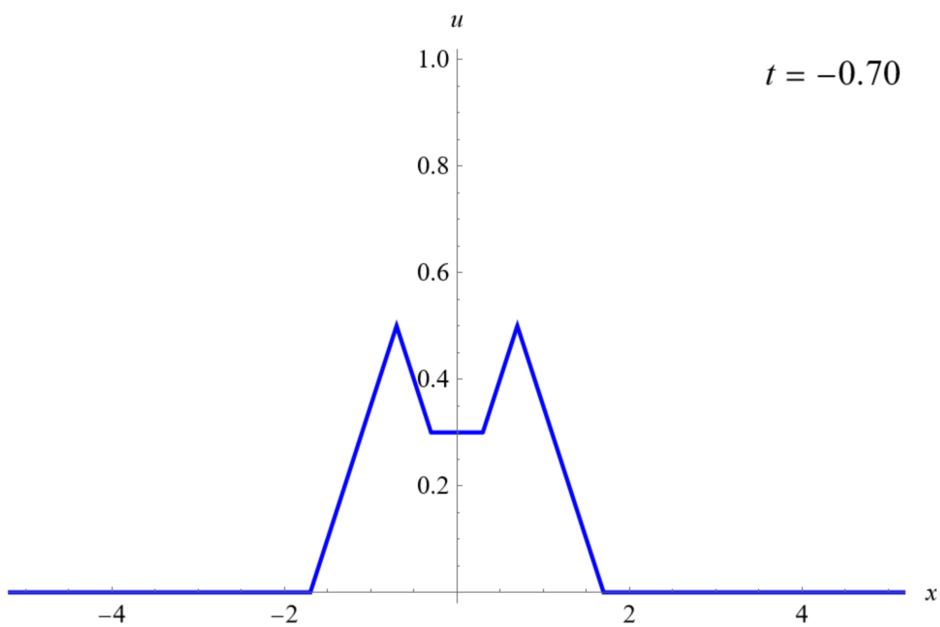



u
1.0
0.8
0.6
0.4
0.2
x
−4
−2
2
4
t = −2

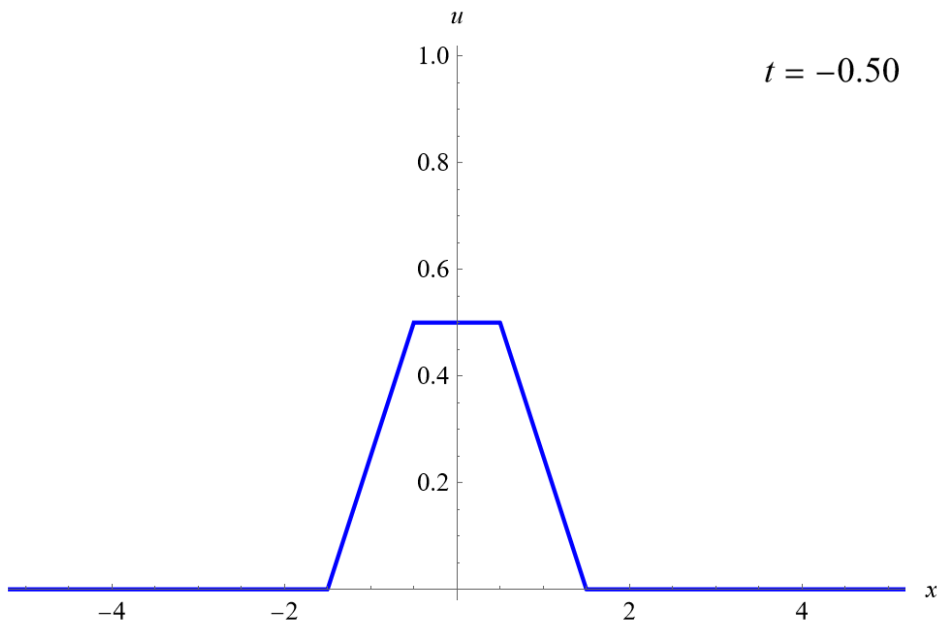

u
1.0
0.8
0.6
0.4
0.2
x
−4
−2
2
4
t = −1



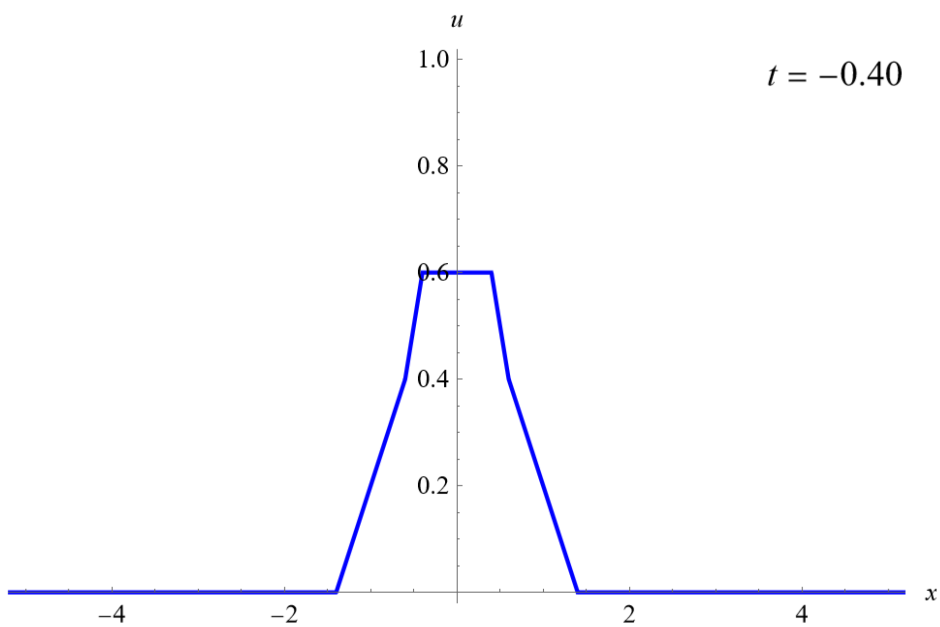
u
1.0
0.8
0.6
0.4
0.2
t = −0.90
x
−4
−2
2
4



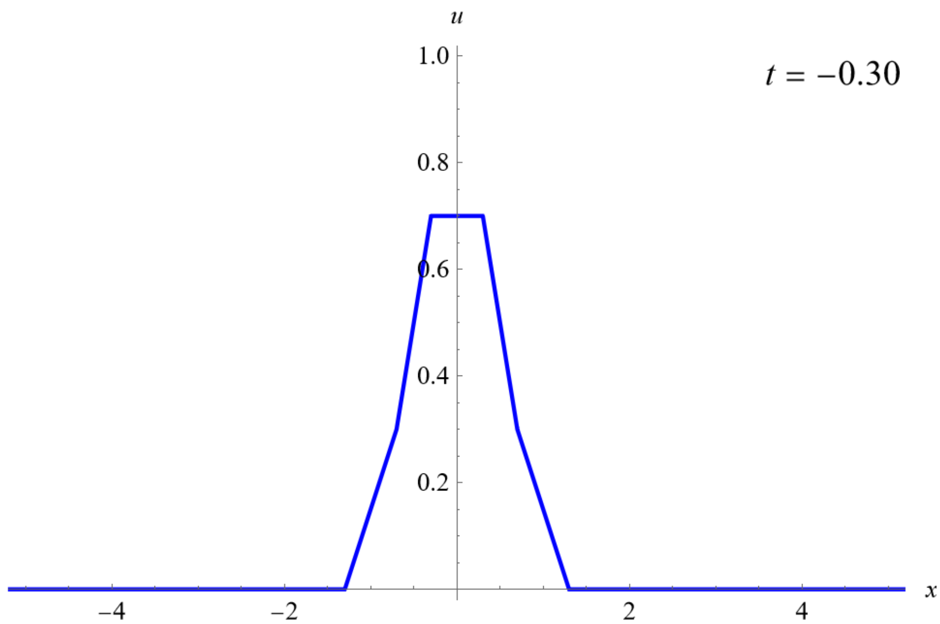
u
1.0
0.8
0.6
0.4
0.2
t = −0.70
x
−4
−2
2
4

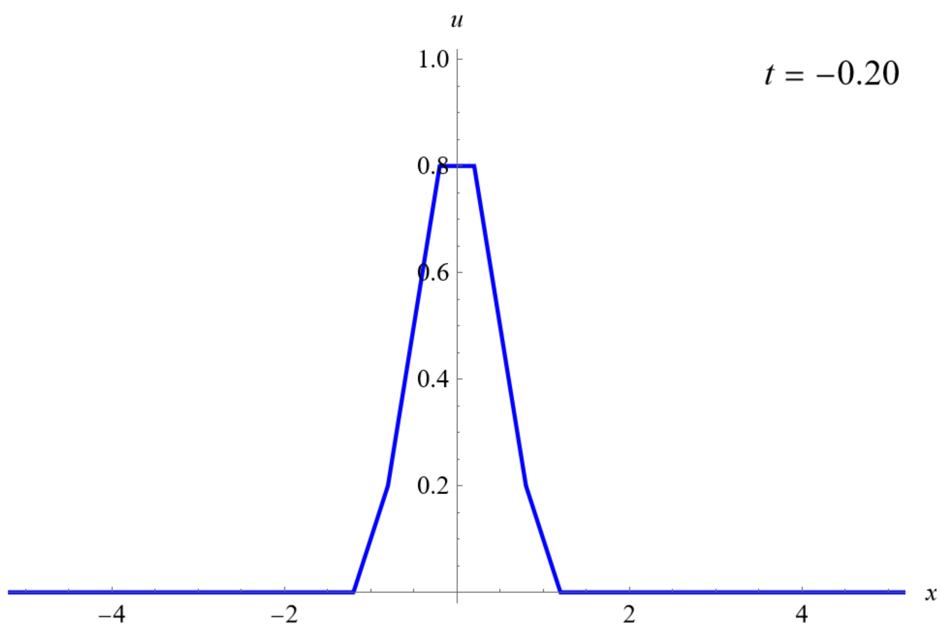


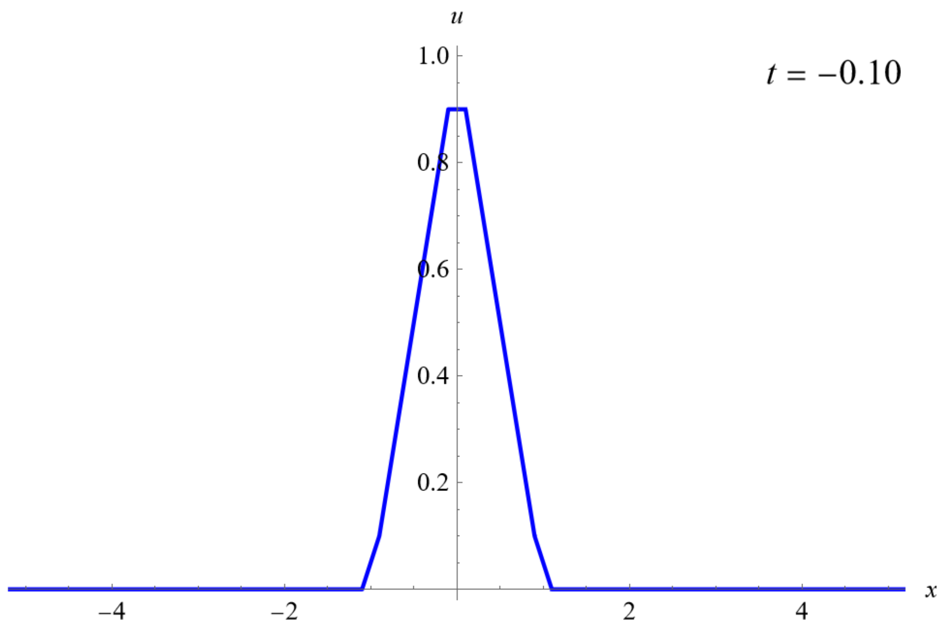
u
1.0
0.8
0.6
0.4
0.2
t = −0.50
−4
−2
2
4
x

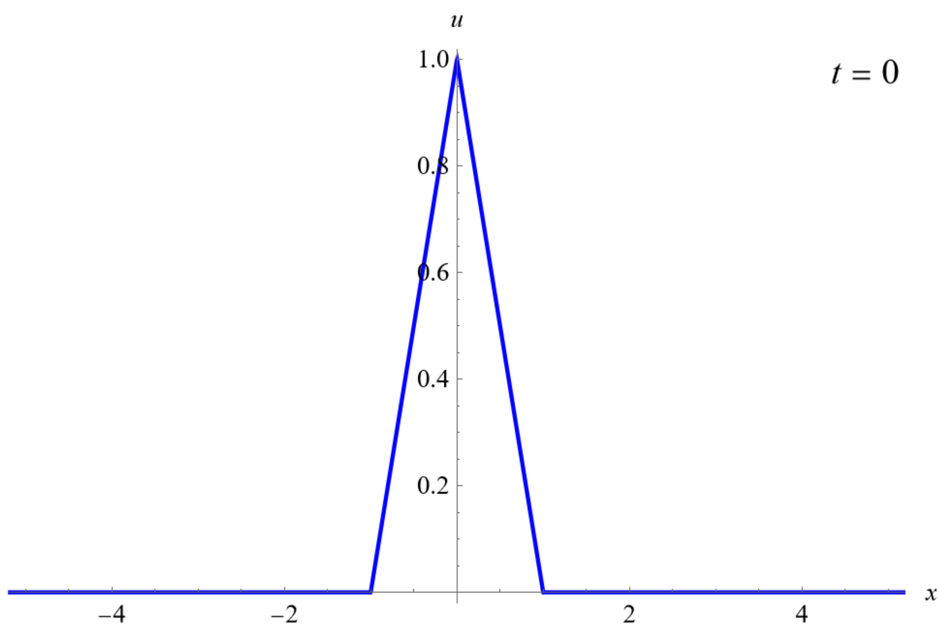


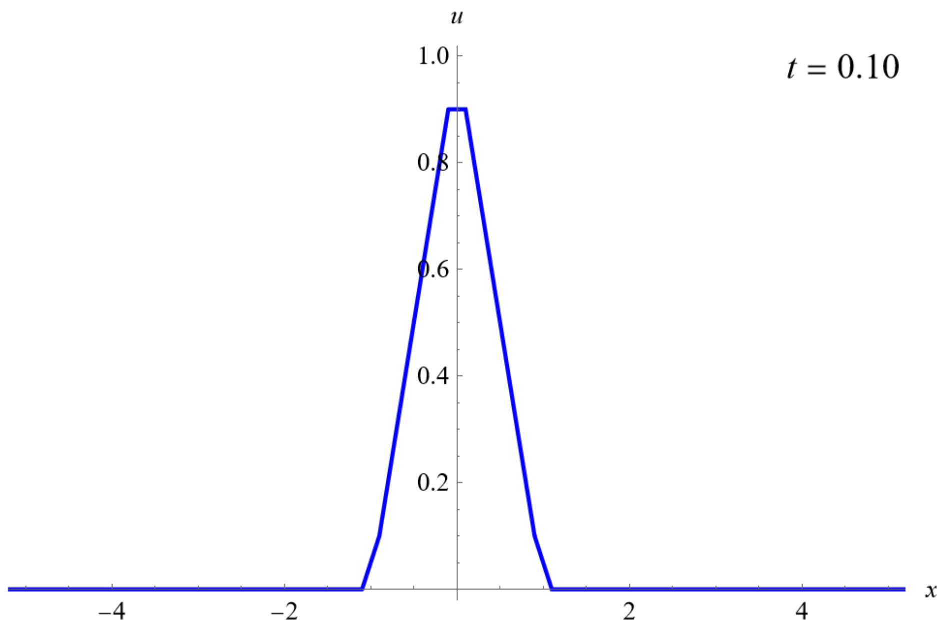
u
1.0
0.8
0.6
0.4
0.2
t = −0.40
−4
−2
2
4
x

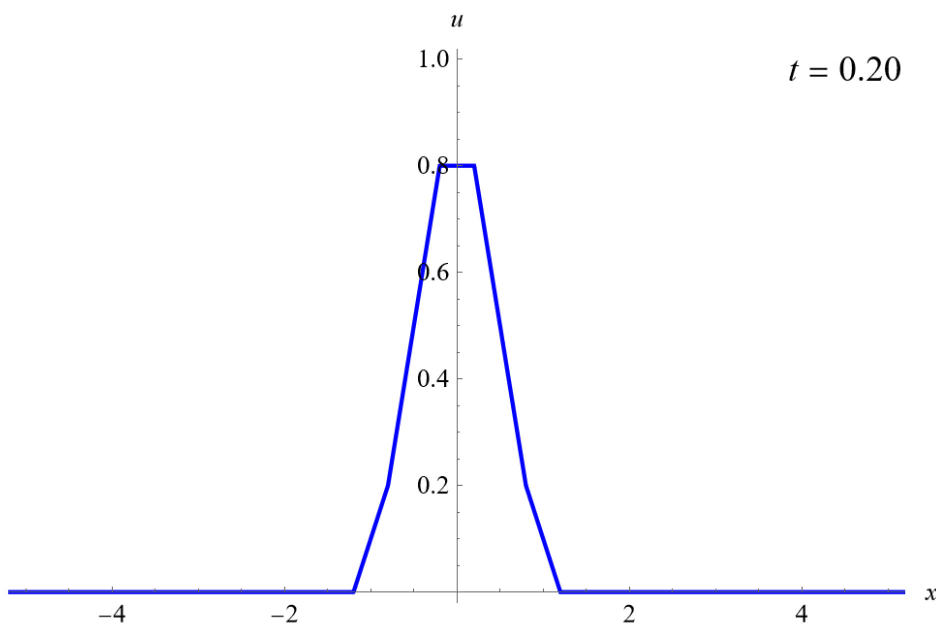

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
t = −0.30

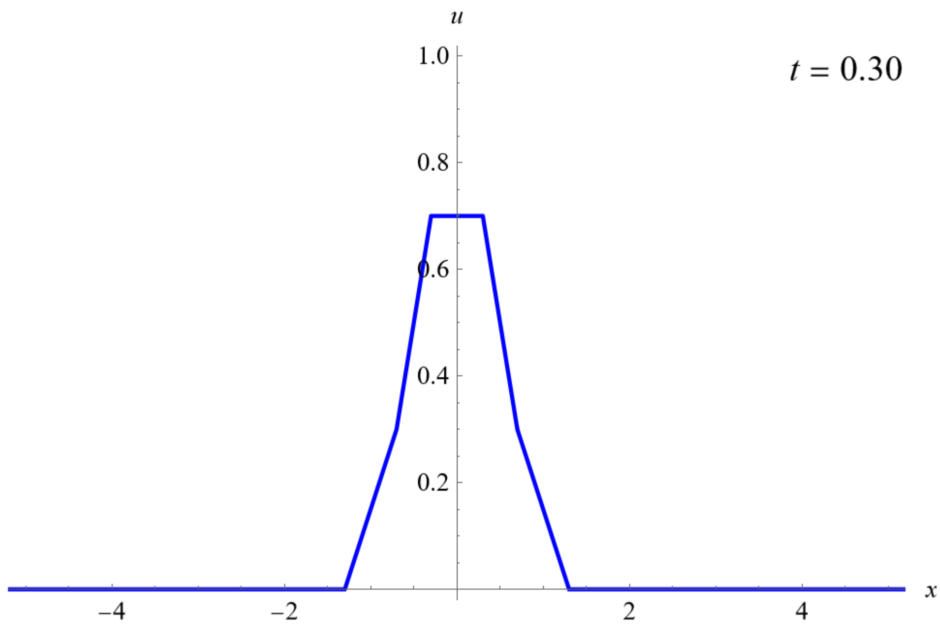

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
t = −0.20

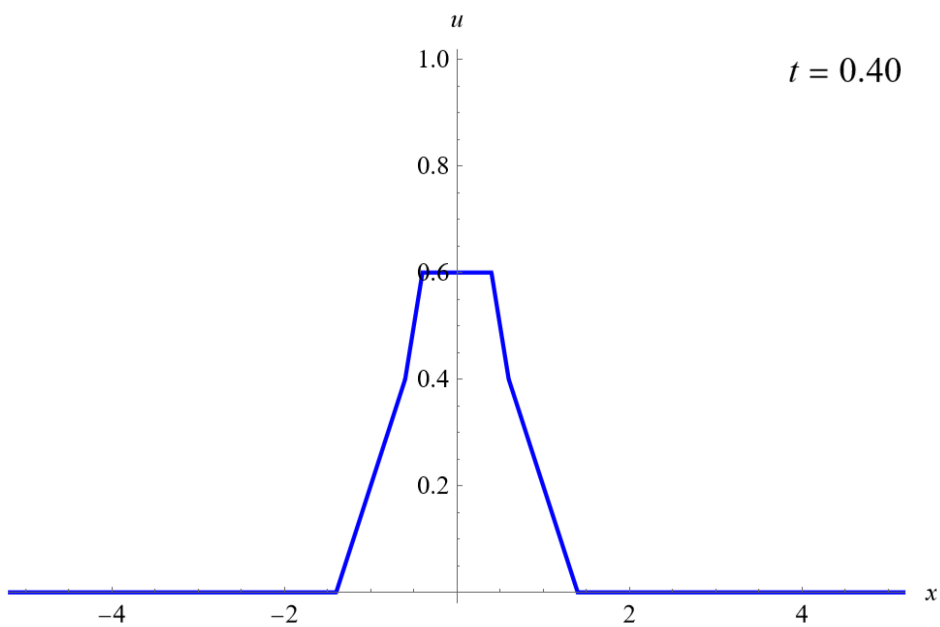

u
1.0
0.8
0.6
0.4
0.2
t = −0.10
−4
−2
2
4
x

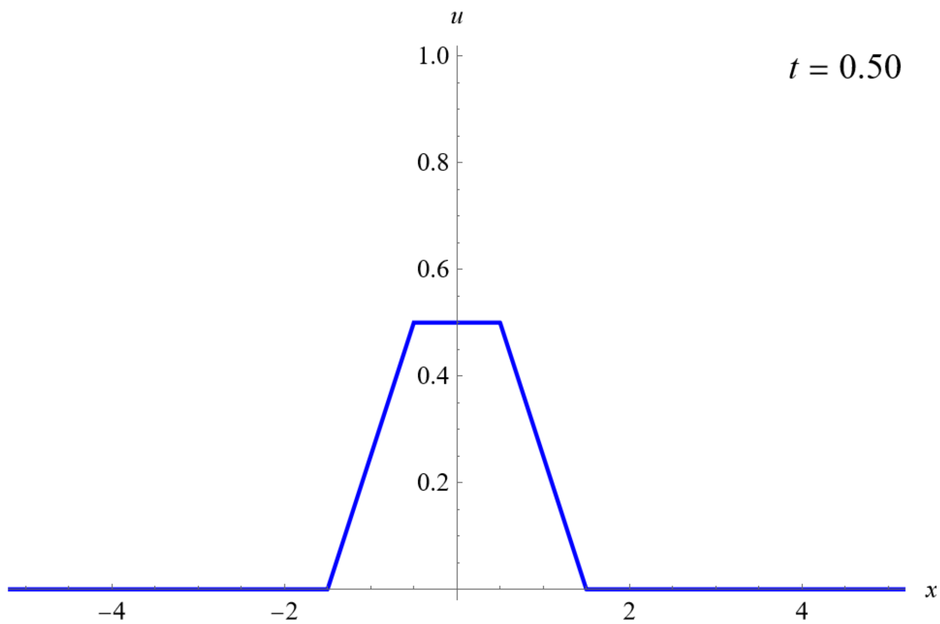

u
1.0
0.8
0.6
0.4
0.2
t = 0
−4
−2
2
4
x

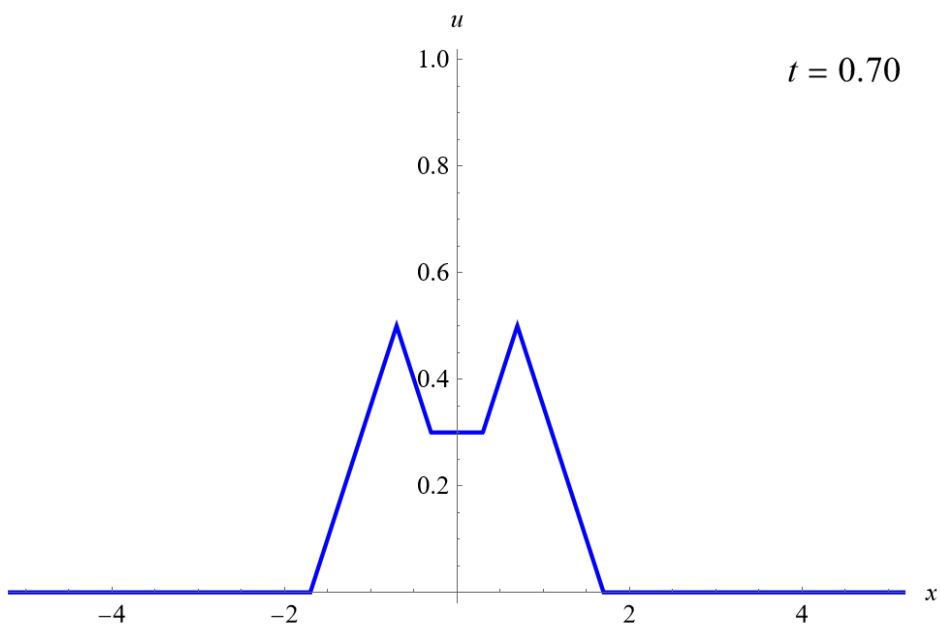

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
$t = 0.10$

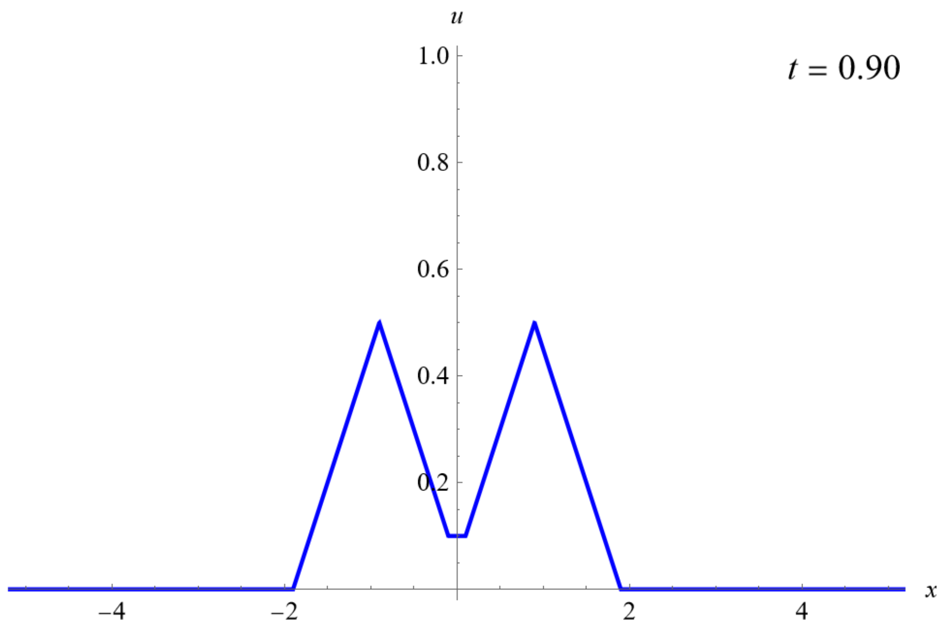

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
$t = 0.20$

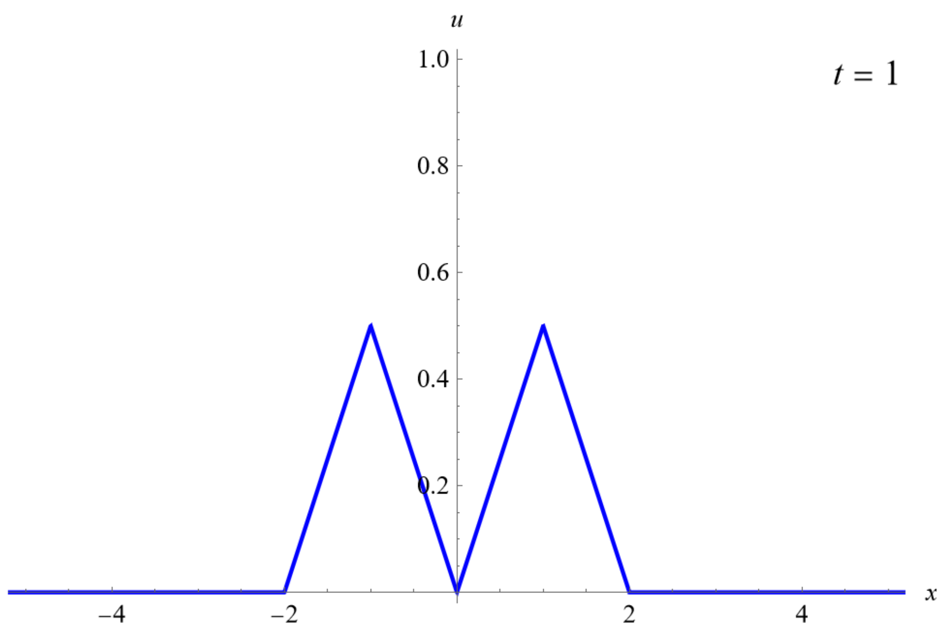

u
1.0
0.8
0.6
0.4
0.2
t = 0.30
−4
−2
2
4
x

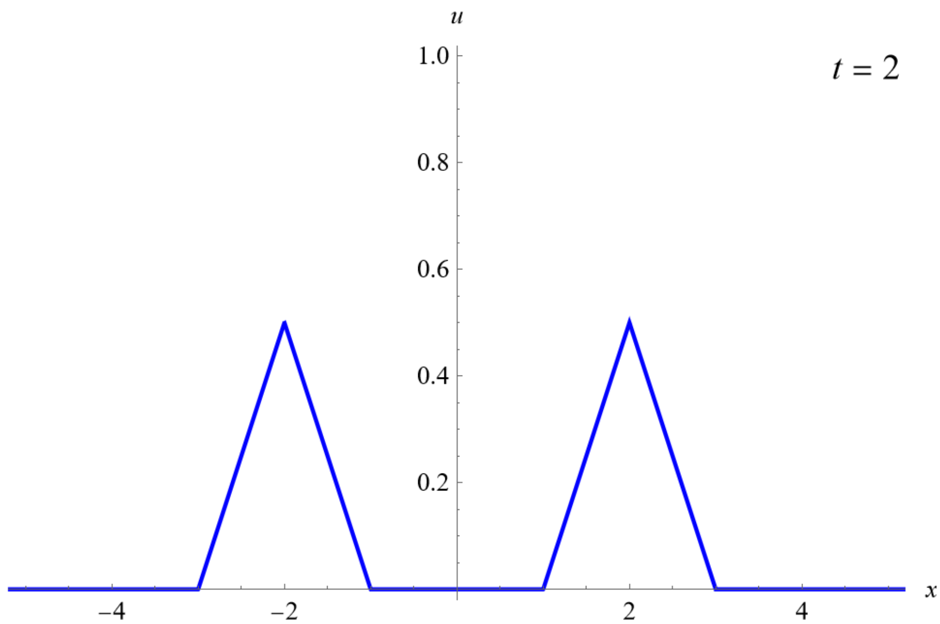

u
1.0
0.8
0.6
0.4
0.2
t = 0.40
−4
−2
2
4
x

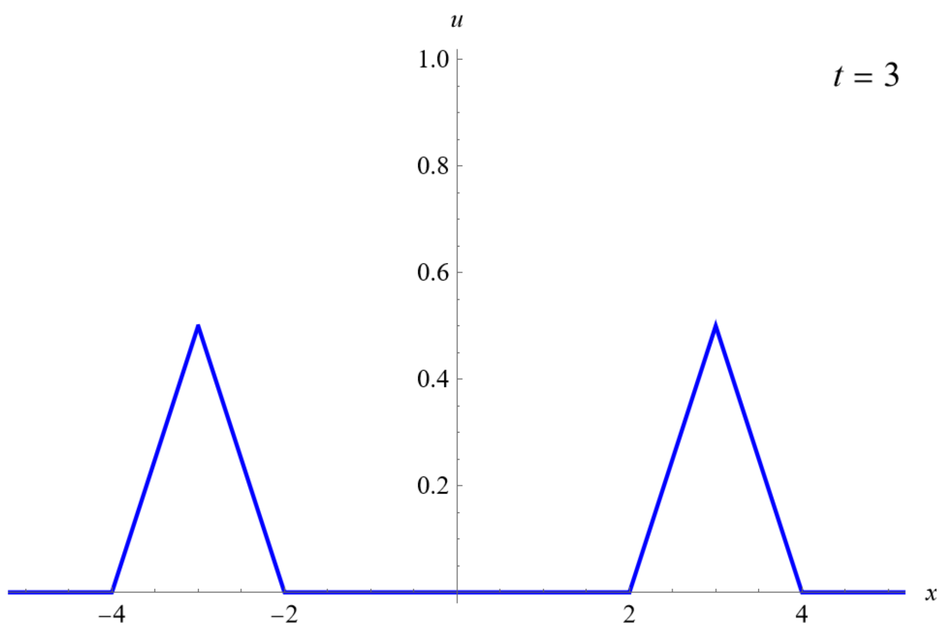

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
t = 0.50

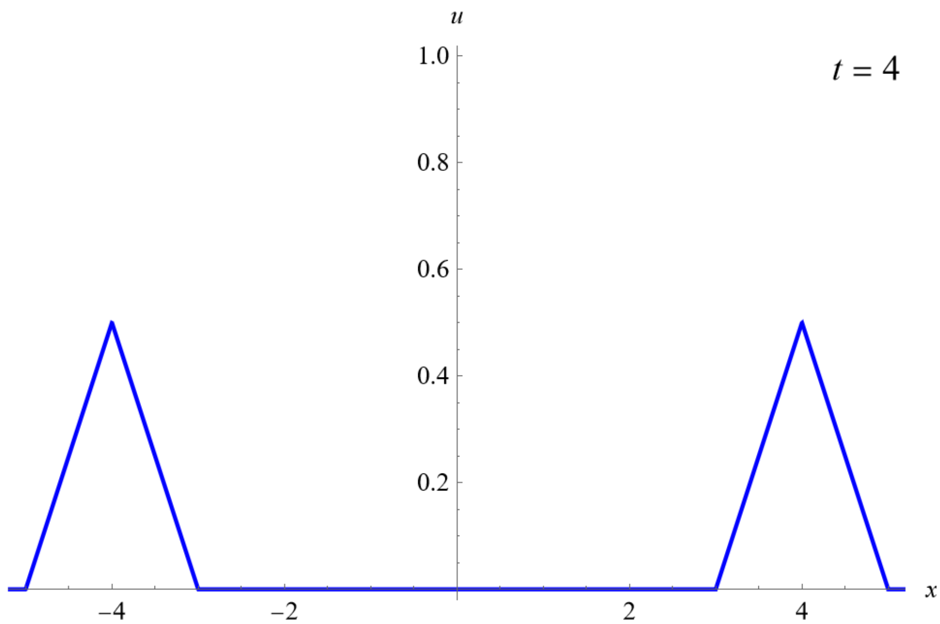

u
1.0
0.8
0.6
0.4
0.2
−4
−2
2
4
x
t = 0.70


u
1.0
0.8
0.6
0.4
0.2
t = 0.90
x
−4
−2
2
4


u
1.0
0.8
0.6
0.4
0.2
t = 1
x
−4
−2
2
4


u
1.0
0.8
0.6
0.4
0.2
t = 2
x
−4
−2
2
4


u
1.0
0.8
0.6
0.4
0.2
t = 3
x
−4
−2
2
4


u
1.0
0.8
0.6
0.4
0.2
t = 4
−4
−2
2
4
x